Willett Action Item: Write a paragraph for Madura on the relationship between optical extinction coefficient and model electrical decay time.

(John Willett, revised 7/31/03)

In a mono-disperse cloud containing N particles per cubic meter, the "electrical-decay time scale," as defined by *Willett and Dye* [2003], is

$$\tau_E \equiv E_0 A_e \varepsilon_0 N/(2eq) \rightarrow E_0 3\pi d^2 \varepsilon_0 N/(8eq)$$

where the "limit" corresponds to spherical particles of diameter, d. τ_E is a reasonable estimate of the time taken for an initially high electric field, E_0, to decay to near zero inside this cloud. (In a real cloud $A_e N$ can be replaced by an integral of the electrical effective area, A_e, over the observed particle-size distribution.)

The optical extinction coefficient in such a cloud [*e.g.*, *Chylek*, 1978] may be written

$$\kappa \equiv A_o N \rightarrow \pi d^2 N/2$$

where A_o is the optical-extinction cross section of a particle, by analogy with A_e above. (In taking the "limit," we have assumed that the cloud particles are much larger than the wavelength of the light in question.) κ gives the optical effective area of the particles per unit volume, with dimensions of inverse meters. Thus, $1/\kappa$ represents unit optical depth in this cloud, the distance over which the intensity of a parallel light beam decays to 1/e of its original value due to scattering and absorption by the particles.

For non-spherical particles, the relationship between τ_E and κ is not simple because of (1) the uncertain relationship between the directions of the light beam and the electrostatic field and (2) the differing dependencies of A_e and A_o on particle shape. (These details can be addressed, for example, by assuming that the cloud particles are prolate spheroids of a given axis ratio, aligned with the electric field, and that the light beam is parallel to the field.) For spherical particles, however, both of these problems disappear, and the two equations above can be combined as follows:

$$\tau_E \rightarrow E_0 3\varepsilon_0 \kappa/(4eq)$$

Thus, the electrical-decay time scale is seen to be directly proportional to the optical extinction coefficient.

One example will suffice for the sake of illustration. In the ABFM anvil dataset, as reported by *Willett* [2003], there are 39 (out of a total of 1608) 30 s flight intervals during which the calculated value of $\tau_E \geq 10{,}000$ s (about 2.8 hours), the maximum value of τ_E being 16657 s. (Some of these high values may be eliminated by the near-convective-core filter that is currently being implemented by NCAR.) Note that 10,000 s is comparable to the standoff time of 3 hours that is specified in the current "Anvil Clouds" rule of the LCC. At an altitude of 10 km ($q \approx 3.0 \times 10^7$ $m^{-3}s^{-1}$) $\tau_E = 10{,}000$ s corresponds to $\kappa \approx$ 0.14/m (or $1/\kappa \approx 6.9$ m), where we continue to use $E_0 = 50$ kV/m. Since such dense anvils invariably have a physical thickness, H, of several km, they satisfy any reasonable definition of "optically thick."

One meaning that has been suggested for the term, optically thick, is that the cloud optical thickness, κH, be 10 or greater. This value can be justified as follows: *Klett* [1972] estimated the thickness of the "screening layer" on an electrified cloud as $h \approx 1.85/(A_e N)$. Using the above relations for τ_E and κ, we have

$$h \approx 1.85\ E_0\varepsilon_0/(2eq\tau_E) \rightarrow 1.85\cdot 4/(3\pi d^2 N) = 1.85\cdot 2/(3\kappa) \approx 1.23/\kappa$$

Thus, the screening-layer thickness is nearly equal to the unit optical depth, $1/\kappa$, in such a cloud. Clearly, significant charge cannot be stored inside a cloud unless its physical thickness is significantly greater than its screening layers -- say, $H/h \sim \kappa H \geq 10$. The table gives some typical values of κH by way of illustration.

ETmScl (s):	100	300	1000	3000	10000
1/kappa (m):	691	230	69	23	7
H (m)		Optical Thickness (non-dimensional)			
100	0	0	1	4	14
300	0	1	4	13	43
1000	1	4	14	43	145
3000	4	13	43	130	434

<u>References</u>

Chylek, P., Extinction and liquid water content of fogs and clouds, *J. Atmos. Sci., 35*, 296-300, 1978.

Klett, J.D., Charge screening layers around electrified clouds, *J. Geophys. Res.*, 77, 3187-3195, 1972.

Willett, J.C., Electrical decay estimates in anvil clouds, Report No. 2 of 2 under Contract No. CC-90233B, 4/11/03.

Willett, J.C., and J.E. Dye, A simple model to estimate electrical decay times in anvil clouds, in Proceedings of the 12th ICAE, Versailles, 9-13 June, 2003.